Reply to "Comment on 'Soliton ratchets induced by excitation of internal modes' "

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In this Reply to the Comment of Quintero *et al.*, we show that the energy current $J(t)$ is a very different physical quantity than the energy current $P(t)$ defined in our original paper [Phys. Rev. E **69**, 056612 (2004)]. Consequently, to assume they are the same physical quantity, as is done in the Comment, renders the Comment devoid of meaning.

The derivation of the energy current equation of motion, Eq. (10), in the Comment [1], namely

$$\dot{J}(t) = -\beta J(t) - \lambda Q f(t)$$

is correct and was derived many years ago by Olsen and Samuelson [2]. The definition of the current $J(t)$ is

$$J(t) \equiv -\int_{-\infty}^{\infty} dx\, \phi_{,x}(x,t)\phi_{,t}(x,t), \tag{1}$$

where $\phi(x,t)$ satisfies the damped driven sine-Gordon (SG) equation

$$\phi_{,tt}(x,t) - \phi_{,xx}(x,t) = -U'[\phi(x,t)] - \beta\phi(x,t) + f(t). \tag{2}$$

In Ref. [3], we defined a completely different energy current $P(t)$ for the damped ac-driven SG equation, namely

$$P(t) \equiv -\int_{-\infty}^{\infty} \sigma_{,t}(\xi)\sigma_{,x}(\xi)dx, \tag{3}$$

where

$$\xi \equiv \Gamma(t)[x - X(t)],$$

and where the solution of the damped ac-driven SG is

$$\phi(x,t) = \sigma(\xi) + \chi(\xi,t).$$

The function $\chi(\xi,t)$ is defined on the phonon states of the SG.

The two energy currents $J(t)$ defined in Eq. (1) and $P(t)$ defined in Eq. (3) are completely different from each other and lead to completely different time-averaged currents. The fact that the infinite time average of $J(t)$, namely $\langle J(t)\rangle$ vanishes has absolutely no relevance to our very different nonvanishing infinite time average $\langle P(t)\rangle$ plotted as a function of the phase angle θ in Fig. 1 of Ref. [3].

$P(t)$ and $J(t)$ are very different physical quantities. Therefore, to treat them as the same physical quantity as is done in the Comment [1] makes their constructed contradiction pointless, and with it the whole comment is devoid of meaning.

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[1] N. R. Quintero, B. Sanchez-Rey, and J. Casado-Pasqual, preceding paper, Phys. Rev. E **71**, 058601 (2005).
[2] O. H. Olsen and M. R. Samuelsen, Phys. Rev. B **28**, 210 (1983).
[3] C. R. Willis and M. Farzaneh, Phys. Rev. E **69**, 056612 (2004).